OCR

Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $x+\frac{1}{1-x}+\frac{2}{1+x}$ as a single fraction, simplifying your answer. **[3]**

2 The points $O(0, 0, 0)$, $A(2, 8, 2)$, $B(5, 5, 8)$ and $C(3, -3, 6)$ form a parallelogram $OABC$. Use a scalar product to find the acute angle between the diagonals of this parallelogram. **[5]**

3 **(i)** Find the first three terms in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, where $|x|<\frac{1}{2}$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $\frac{x+3}{\sqrt{1-2x}}$. **[2]**

4 Show that $\int_0^{\frac{1}{4}\pi} \frac{1-2\sin^2 x}{1+2\sin x\cos x}\,dx = \frac{1}{2}\ln 2$. **[5]**

5 The equations of three lines are as follows.

Line A: $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \mathbf{k} + s(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

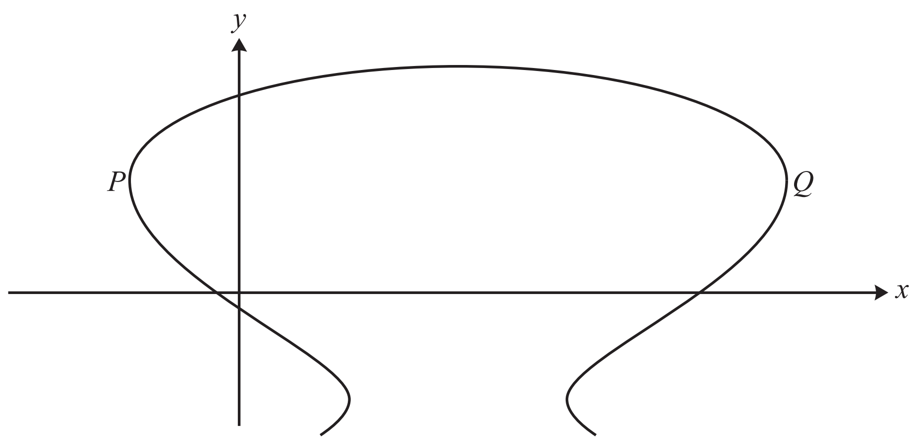
Line B: $\mathbf{r} = 2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} + t(\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$

Line C: $\mathbf{r} = -\mathbf{i} + 19\mathbf{j} + 15\mathbf{k} + u(2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k})$

(i) Show that lines A and B are skew. **[4]**

(ii) Determine, giving reasons, the geometrical relationship between lines A and C. **[2]**

6



The diagram shows the curve with equation $x^2+y^3-8x-12y=4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**

7 A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. **[5]**

(ii) Find the cartesian equation of the curve. **[3]**

(iii) State the set of values that x can take and hence sketch the curve. **[3]**

8 **(i)** Use division to show that $\frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}$. **[3]**

(ii) Find $\int_1^2 6t^2 \ln(t+2)\,dt$. Give your answer in the form $A + B\ln 3 + C\ln 4$. **[6]**

9 Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,dx = \frac{1}{2}\ln\frac{3}{2} + \frac{1}{3}$. **[9]**

10 A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of $0.01\,\text{m}^3$ per second. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]

(i) Show that $h^2 \frac{dh}{dt} = -\frac{9}{400\pi}$. **[5]**

(ii) Express h in terms of t. **[4]**

(iii) Find the time it takes to empty the container, giving your answer to the nearest minute. **[2]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$x(1-x^2)+(1+x)+2(1-x)$ oe	M1	condone one sign error	if M0B0, SC1 for any pair of terms correctly combined into a single fraction, may be unsimplified
			$1-x^2$ oe	B1	any correct denominator common to all three fractions	
			$\dfrac{3-x^3}{1-x^2}$ oe cao	A1	must be fully simplified; mark the final answer	eg $\dfrac{x(3-x^3)}{x(1-x^2)}$ oe may score a maximum of M1B1A0
				[3]		
2			± ((3 – 2)**i** + (− 3 − 8)**j** + (6 – 2)**k**) soi	B1	NB **i** – 11**j** + 4**k**	or
			their ± (**i** – 11**j** + 4**k**). ±(5**i** + 5**j** + 8**k**) both diagonals used ; evaluation not essential	M1	if M0 SC2 for 84° (or 84.5°), or 52(.3°) or 39° or (38.5° or 43(.2°) or 46(.0°) found from scalar product or SC1 for the equivalent obtuse angle	B3 for correct use of Cosine Rule (using the midpoint of the diagonals of the parallelogram) $[\cos\theta]=\dfrac{34.5+28.5-72}{2\sqrt{34.5}\sqrt{28.5}}$ oe
			$\pm(1\times5+(-11)\times5+4\times8)=\sqrt{1^2+11^2+4^2}\times\sqrt{5^2+5^2+8^2}\cos\theta$ oe	A1	must be fully correct	
			$\theta=\cos^{-1}\dfrac{\pm18}{\sqrt{138}\times\sqrt{114}}$	A1		
			81.7 to 82°	A1 **[5]**	1.4 to 1.43 rad	B2 for 81.7 to 82° unsupported or B3 + B2 possible for Cosine Rule

Question			Answer	Marks	Guidance	
3	(i)		$1+(-\frac{1}{2})(-2x)+(-\frac{1}{2})(\frac{-3}{2})\frac{(\pm 2x)^2}{2!}[+...]$	B1 B1	first two terms third term	allow recovery from omission of brackets do not allow $2x^2$ unless fully recovered in answer
			$1+x+\frac{3}{2}x^2$ oe	B1 **[3]**		
	(ii)		use of $(x+3)\times$ their$(1+x+\frac{3}{2}x^2)$	M1		
			coefficient is 5.5 oe	A1 **[2]**	or **B2** www in either part	may be embedded (eg $5.5x^2$ alone or in expansion)
4			$\int \frac{\cos 2x}{1+\sin 2x}\,(\mathrm{d}x)$	B1* B1*	$\cos 2x = 1-2\sin^2 x$ or $(1+)\sin 2x = (1+)\ 2\sin x\cos x$ seen numerator and denominator both correct in the integral soi	if B0B0M0A0, SC4 for $F[x] = ½\ln(1+2\sin x\cos x)$ or $½\ln(1+\sin 2x)$ final mark may still be awarded
			$F[x]= k\ln(1+\sin 2x)$ soi	M1dep*	or $k\ln(1+u)$ or $k\ln(u)$ following their substitution www	
			$k = ½$	A1	correct k for their substitution	
			$½\ln(1+\sin(\pi/2)) - ½\ln(1+0)$ $= ½\ln 2$	A1 AG **[5]**	correct use of limits www	minimum working: $½\ln 2 - ½\ln 1$ or $½\ln(1+1)$ oe

Question			Answer	Marks	Guidance	
5	(i)		$1 - s = 2 + t$ $4 + 2s = 8 + 3t$ $1 + 2s = 2 + 5t$	B1	for all three equations NB third equation may appear later, or with values already substituted	or M1 for one value (of s or t) found from one pair of equations
			value of either s or t obtained from valid method	M1	eqns (i) and (ii): $s = 0.2$, $t = -1.2$	A1 for substitution of this value (of s or t) in third equation and obtaining the other parameter (ie of t or s);
			correct pair of values	A1	eqns (i) and (iii): $s = {}^{-4}/_{7}$, $t = {}^{-3}/_{7}$ eqns (ii) and (iii) $s = 4.25$, $t = 1.5$	NB (0.2, − 0.12) or $({}^{-4}/_{7}, {}^{-12}/_{7})$ or (4.25, − 5.25) if s found first and (– 2.5, –1.2) or $({}^{19}/_{14}, {}^{-3}/_{7})$ or (– 2.5, 1.5) if t found first
			eg $1 + 2\times0.2 \neq 2 + 5\times-1.2$ oe isw NB A0 for $1 + 2\times0.2 = 2 + 5\times-1.2$ unless clarified by suitable comment	A1	correct substitution of correct values in correct equation	**or** find same parameter from second pair of equations
				[4]		A1 for correct demonstration of inconsistency NB clear statement needed if two different values of same parameter found
5	(ii)		$2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k} = -2(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ oe	B1	allow equivalent in words, but scale factors must be correct	eg direction of A is $-½\times$direction of C
			eg line A goes through (1, 4, 1) but line C goes through (1, 15, 11), so they do not coincide so the lines are parallel eg demonstration of different y or z values on each line for (say) $x = 1$, so lines are parallel	B1		
				[2]		

Question			Answer	Marks	Guidance	
6			$3y^2\frac{dy}{dx}$	B1	or $2x\frac{dx}{dy}$	if B0B0 M0
			$2x-12\frac{dy}{dx}-8$	B1	$3y^2-8\frac{dx}{dy}-12$	SC2 for $\frac{dy}{dx}=$
			their $3y^2\frac{dy}{dx}-12\frac{dy}{dx}=8-2x$ soi must be two terms on each side and must follow from RHS $=0$	M1	their $2x\frac{dx}{dy}-8\frac{dx}{dy}=-3y^2+12$ must be two terms on each side must follow from RHS $=0$	$\frac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x+8+12\frac{dy}{dx})$ M1 may be earned for setting correct denominator equal to 0
			$\frac{dy}{dx}=\frac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\frac{dx}{dy}=0$ is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$	
			their $3y^2-12=0$	M1*		$x\neq 4$ not required
			$y=(\pm)\ 2$	A1	A0 if $\frac{dy}{dx}$ incorrect	
			substitution of their positive y value in original equation	M1dep*		ignore substitution of -2
			$x=10$, $x=-2$ and no others cao	A1 **[8]**	A0 if $\frac{dy}{dx}$ incorrect	condone omission of formal statement of coordinates (10, 2) and (–2, 2)

Question			Answer	Marks	Guidance	
7	(i)		$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of $x = 2\sin t$
			$\frac{dy}{dx} = \text{their}\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
			$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
			$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to $1 - 2\sin t$ www	A1	or equivalent intermediate step	
			(1, 1½)	B1 **[5]**	NB $t = \frac{\pi}{6}$	from $1 - 2\sin t = 0$
7	(ii)		$(y =)\ 1 - 2\sin^2 t + 2\sin t$	B1	may be awarded after correct substitution for x eg $(y =)\ 1 - {}^{x^2}/_4 - \sin^2 t + 2\sin t$	or $(y =)\ x + \cos 2t$
			substitution of $\sin t = ½x$ to eliminate t	M1		substitution of $t = \sin^{-1}({}^x/_2)$ to eliminate t
			$y = 1 + x - ½x^2$ oe isw	A1 **[3]**	or B3 www	$y = x + \cos 2(\sin^{-1}({}^x/_2))$ oe isw

Question			Answer	Marks	Guidance	
7	(iii)		$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \le 2$ or $\lvert x \rvert \le 2$	B1	cao	
			sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	
			positive y-intercept and one distinguishing feature isw	A1		one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), y – intercept is (0, 1), x-intercept is $(1 - \sqrt{3}, 0)$
				[3]		
8	(i)		t^2 in quotient and $t^3 + 2t^2$ seen	B1	or $\dfrac{t(t^2-4)+4t}{(t+2)}$	or $\dfrac{(t+2)^3 - 6t^2 - 12t - 8}{(t+2)}$
			$-2t$ in quotient and $-2t^2 - (-2t^2 - 4t) = 4t$ seen	B1	$\dfrac{t(t+2)(t-2)}{(t+2)} + \dfrac{4t}{t+2}$	$\dfrac{(t+2)^3}{(t+2)} - \dfrac{6((t+2)^2 - 4t - 4) + 12t + 8}{(t+2)}$ oe
			completion to obtain correct quotient and remainder identified www	B1	$t(t-2) + \dfrac{4(t+2)-8}{t+2}$	$(t+2)^2 - 6(t+2) + \dfrac{12t+16}{t+2}$ oe $= t^2 + 4t + 4 - 6t - 12 + \dfrac{12(t+2)-8}{t+2}$ oe both steps needed for final B1
				[3]		
8	(i)		alternatively $\dfrac{t^3}{t+2} \equiv At^2 + Bt + C + \dfrac{D}{(t+2)}$	B1	or $t^3 \equiv (At^2 + Bt + C)(t+2) + D$	or B1 for $\dfrac{t^2(t+2) - 2t^2}{(t+2)}$
			equate coefficients to obtain correctly $A = 1$, $0 = 2A + B$ and $B = -2$ www	B1		B1 for $t^2 + \dfrac{-2t(t+2)+4t}{(t+2)}$
			$0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www	B1		B1 for $t^2 - 2t + \dfrac{4(t+2)-8}{(t+2)}$
				[3]		

Question			Answer	Marks	Guidance	
8	(ii)		integration by parts with $u = \ln(t+2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt)$	M1*	$f(t)$ must include t^3 and $g(t)$ must **not** include a logarithm	ignore spurious dx etc
			$2t^3\ln(t+2) - \int \frac{2t^3}{t+2}(dt)$ cao	A1		alternatively, following $u = t + 2$
			result from part (i) seen in integrand; must follow award of at least first M1	M1*	no integration required for this mark	$\int 2(u^2 - 6u + 12 - \frac{8}{u})du$ oe
			$F[t] = 2t^3\ln(t+2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16\ln(t+2)$	A1	$2t^3\ln(t+2) - \frac{2t^3}{3} + 2t^2 - 8t + 16\ln(t+2)$	$\frac{2u^3}{3} - 6u^2 + 24u - 16\ln u$ and $2t^3\ln(t+2)$
			their $F[2] - F[1]$	M1dep*	at least one of their terms correctly integrated	NB limits following substitution are $u = 4$ and $u = 3$
			$-6⅔ - 18\ln3 + 32\ln4$ oe cao	A1 **[6]**		
9			$\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ may be seen in later work	B1	or $\frac{A}{1+2x} + \frac{Bx+C}{(1-x)^2}$ may be seen later in later work	if B0M0, SC1 for $\frac{1}{1+2x}$ seen
			$2 + x^2 \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$	M1	or $A(1-x)^2 + (Bx + C)(1 + 2x)$	allow only sign errors , not algebraic errors
			$A = 1$, $B = 0$ and $C = 1$ www	A1A1A1		
			$\int \left(\frac{1}{1+2x} + \frac{1}{(1-x)^2}\right)dx =$ $a\ln(1+2x) + b(1-x)^{-1}$	M1*	a and b are non-zero constants	ignore extra terms
			$F(x) = ½\ln(1 + 2x) + (1 - x)^{-1}$	A1		
			their $\frac{1}{2}\ln(\frac{3}{2}) + \frac{4}{3} - (\frac{1}{2}\ln1 + 1)$	M1dep*		

Question			Answer	Marks	Guidance	
			$\frac{1}{2}\ln(\frac{3}{2})+\frac{4}{3}-0-1$	A1 **[9]**	and completion to given result www	NB $\frac{1}{2}\ln(\frac{3}{2})+\frac{1}{3}$
10	**(i)**		$\frac{dV}{dt}=\pm 0.01$	B1		
			by similar triangles, $\frac{h}{4.5}=\frac{r}{3}$	B1	may be implied by $r=\frac{2h}{3}$ oe	
			$\frac{dV}{dh}=\frac{4}{9}\pi h^2$ oe	B1		
			$\frac{dh}{dt}=\pm 0.01\times$ their $\frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either r or h or both
			$-0.01=(\frac{4}{9}\pi h^2)\times\frac{dh}{dt}$	A1 **[5]**	completion to given result www	$h^2\frac{dh}{dt}=\frac{-0.09}{4\pi}=\frac{-9}{400\pi}$
10	**(ii)**		$\int h^2 dh=\int\frac{-9}{400\pi}dt$ oe soi	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
			$\frac{h^3}{3}=\frac{-9}{400\pi}t(+c)$	A1		
			substitution of $t=0$ and $h=4.5$ in their expression following integration	M1	expression must include c and powers must be correct on each side	
			$h=\sqrt[3]{\frac{729}{8}-\frac{27t}{400\pi}}$ oe isw	A1 **[4]**	allow – 0.0215 or – 0.02148591…r.o.t to 4 sf or more and similarly 91.125	$91.125={}^{729}/_8$
10	**(iii)**		set $h=0$ and solve to obtain positive t	**M1**	or $(t=)\frac{1}{3}\pi\times 3^2\times 4.5\div 0.01$ $(=1350\pi)$	NB $1350\pi=4241.150082\ldots$
			71 minutes cao	**A1** **[2]**		